

On classes of C3 and D3 modules

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Abstract

© 2018, Hacettepe University. All rights reserved. This paper aims to study the notions of A-C3 and A-D3 modules for some class A of right modules. Several characterizations of these modules are provided and used to describe some well-known classes of rings and modules. For example, a regular right R-module F is a V-module if and only if every F-cyclic module is an A-C3 module, where A is the class of all simple right R-modules. Moreover, let R be a right artinian ring and A, a class of right R-modules with a local ring of endomorphisms, containing all simple right R-modules and closed under isomorphisms. If all right R-modules are A-injective, then R is a serial artinian ring with $J_2(R) = 0$ if and only if every A-C3 right R-module is quasi-injective, if and only if every A-C3 right R-module is C3.

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Keywords

A-C3 module, A-D3 module, V-module

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